Effect of a global plume distribution on Earth normal modes

Philippe Lognonné, Barbara Romanowicz

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EFFECT OF A GLOBAL PLUME DISTRIBUTION ON EARTH NORMAL MODES

Philippe Lognonne and Barbara Romanowicz

Institut de Physique du Globe

Abstract: We show that a global distribution of small scale heterogeneities in the lower mantle induces both frequency splitting and amplitude scatter of low angular order modes. These effects can be used in order to map the global distribution of small scale structures in the mantle, such as plumes. Two independent datasets, consisting respectively of lower mantle modes interaction coefficients and of observed amplitudes of core modes, can then be used to constrain the low angular degrees of the spatial distribution and the mean size of heterogeneities, allowing us to constrain the mantle structure at different scales. If these heterogeneities are assumed to be plumes associated with hotspot activity, the mean size of plumes obtained using the two datasets is then in good agreement with that obtained from geodetical or numerical convection studies.

Introduction

Observed splitting of some low angular order normal modes has been interpreted in terms of 3D perturbations to P and S velocities in the lower mantle [*Ritzwoller et al*, 1988; *Giardini et al*, 1988], complementing the models obtained by inversion of P travel time residuals [*Dziewonski*, 1984]. However, models obtained from splitting data require unusual scaling relations between Vp, Vs, ρ, and/or CMB undulations [*Ritzwoller et al*, 1988] and are based on the assumption that modes are sensitive only to smooth lateral variations and coupling effects between modes can be neglected. But, as noted by *Neele et al* [1989], small scale heterogeneities induce coupling effects on low angular order modes, and, if such heterogeneities are present, their spatial distribution may contain a significant smooth component, which may contribute to tomographic maps in addition to the effects of large scale convective flows. In this paper, we show that a global distribution of small scale lower mantle heterogeneities superimposed on a smooth convection pattern can explain not only splitting observations but also the scatter in some recent low angular mode amplitude observations. We consider a distribution of hotspot plumes, whose correlation with mantle models, as well as geoid and surface topography, has been studied by *Richards et al* [1988] and *Cazenave et al* [1989]. We show that it is possible to explain splitting data with more than 80% variance reduction and to separate the smooth tomographic model of the mantle into a part associated with the plumes distribution and another associated with the global convective pattern.

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The plume model.

Since the models of *Wilson* [1963] and *Morgan* [1971], it has been proposed that plumes rise right through the mantle, creating hotspot volcanoes at the Earth's surface. They could be generated by thermal instabilities located in the D" layer [*Olson et al*, 1987], in which there is evidence for strong and sharp lateral variations [*Vinnik et al*, 1989]. They could propagate in cylindrical conducts with a radius of a few hundred kms and a temperature contrast between 300K and 600K [*Schubert et al*, 1989]. While the number of such instabilities in the earth's mantle is still unknown, it can be related to the number of observed hotspots, which differs between different studies, ranging from 47 in *Richards et al* (1988) to 115 in *Burke & Wilson* [1976]. We have taken a global distribution of plumes located beneath the *Richards et al* [1988] hotspots. Each plume is associated with a cylindrical conduct through the mantle, with a radius of 175 km in the lower and 50 km in the upper mantle. However, since modes are sensitive to the volume of the plume heterogeneity, we have neglected the plume structure in the upper mantle. The temperature contrast of all these plumes is 400K and the thermal expansivity α is 3 x 10^{-5}.

Splitting matrices and modes

The normal modes of a laterally heterogeneous Earth are solutions of the normal mode equation:

$$\omega^2 \mathbf{u} = \mathbf{H}\mathbf{u} \qquad (1)$$

where ω is the eigenfrequency associated with the eigenmode $\mathbf{u}$, and $\mathbf{H}$ the elastodynamic operator. The interaction matrix between two modes with angular degree ℓ and $\ell\prime$ due to only one plume located at the North pole is given by *Lognonné & Romanowicz*,[1990]:

$$\mathbf{H}^{\ell\ell\prime}_{mm\prime}(0) = \sum_{NN\prime=-2}^{NN\prime=2} \int_{\Sigma} Y_\ell^{Nm} X_{\ell\ell\prime}^{NN\prime} Y_{\ell\prime}^{N\prime m\prime} d\Sigma, \qquad (2)$$

where $X_{\ell\ell\prime}^{NN\prime}(\theta,\phi)$ depends on the radially integrated lateral heterogeneities and $Y_\ell^{Nm}(\theta,\phi)$ is the generalized spherical harmonics [*Phinney & Burridge*, 1974]. Matrix (2) can be computed starting from the 3D plume model, using forward and backward Legendre Transformations [*Lognonné & Romanowicz*, 1990]. Let us now consider another plume i located in geographical coordinates Θ_i, Φ_i. Using the

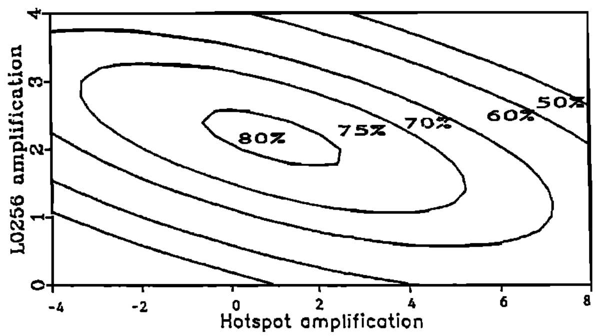

Fig. 1. Map of the χ^2 variance reduction for the interaction coefficients of 28 lower mantle modes [*Ritzwoller et al*, 1988]. The lower mantle model is a superposition of smooth heterogeneities correlated with model L0256 and of a global distribution of plumes. Modes are ${}_0S_{3-9}$, ${}_1S_{3-9}$, ${}_2S_{3-8}$, ${}_3S_1$, ${}_4S_3$, ${}_5S_{3-7}$. The maximun variance reduction is obtained for a L0256 amplification of 2.17 and for a plume temperature contrast of 360K.

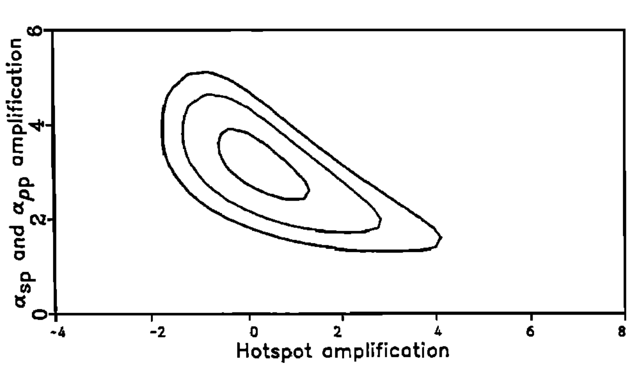

Fig. 2. Map of the χ^2 variance reduction for the same modes as for figure 1, but assuming that V_p heterogeneities of the lower mantle are those of L0256 plus those of plumes, and that the correlations between V_s, V_s and ρ can be amplified.

composition rules of spherical harmonics [*Vilenkin*, 1968] and changing the integration variables in (2) to a new local spherical coordinate system around each plume, the interaction term for all plumes can be written:

$$\mathbf{H}^{\ell\ell'}_{mm'} = \sum_i \mu_i \sum_{kk'} t^{km*}_{\ell}(\Theta_i, \Phi_i, \Psi_i)\mathbf{H}^{\ell\ell'}_{kk'}(0)t^{k'm'}_{\ell'}(\Theta_i, \Phi_i, \Psi_i), \quad (3)$$

where the function $t^{km}_{\ell}(\Theta, \Phi, \Psi)$ is given by *Vilenkin* [1968]

$$t^{km}_{\ell}(\Theta, \Phi, \Psi) = Y^{mk}_{\ell}(\Theta, \Phi)\exp(-\imath m\Psi)$$

Here Ψ_i is taken equal to zero as anisotropy is neglected, and μ_i is a weight depending on the plume i, equal for all plumes in this study. The computation of the eigenmodes can now be done, diagonalizing the matrix (3) using perturbation theory [*Lognonnè & Romanowicz*, 1990].

Constraining the lower mantle with splitting coefficients

The first test, in order to constrain the size of the plume model, is to use the observed splitting coefficients of the lower mantle modes [*Ritzwoller et al*, 1988], sensitive to the smooth part of lateral heterogeneities. We will assume that actual lower mantle heterogeneity corresponds to the superposition of a large scale convective pattern and a distribution of small scale plumes. The lower mantle model L0256 [*Dziewonski*, 1984] correlates well with hotspot distribution at degree 2 but not at degree 6 [*Richards et al*, 1988] so that it is reasonable to assume that L0256 only partially resolves the plume structure and to ask how much it needs to be modified to explain the splitting observations. We thus consider the lower mantle model as a linear combination of L0256 and the plume model. In terms of splitting matrices of a given mode, this can be written as

$$\mathbf{H}_{M84} + x({}_p\mathbf{H}_{L0256} + z_{\rho s}\mathbf{H}_{L0256}) + y({}_p\mathbf{H}_{plumes} + z_{\rho s}\mathbf{H}_{plumes}), \quad (4)$$

where the upper mantle model considered is M84 [*Woodhouse & Dziewonski*, 1984]. Indeces ρ, p, s respectively refer to ρ, V_p and V_s heterogeneity. These splitting matrices are calculated with the same standard scaling relations $\frac{dlogV_p}{dlogV_s} = 0.8$ and $\frac{dlog\rho}{dlogV_p} = .5$ [*Giardini et al*, 1988; *Ritzwoller et al*, 1988]. Hence in (5), the unknown coefficient x is related to possible amplification of model L0256 required in the frequency range of mantle modes, y to an amplification of the plumes, as measured by the quantity $r^2\Delta T$, where r is the radius of the plumes and ΔT the temperature contrast, since low angular modes see the plumes as Dirac functions, and z is related to an increase of the V_p, V_s and ρ correlation ratios. Note that the ratio between x and y also depends on the resolution power of L0256 for lower mantle plumes, since the corresponding terms are not independent. A small y value will thus indicate that L0256 either resolves the low degrees of the lower mantle plume distribution or that there are no plumes in the lower mantle. On the other hand, if x is small, y will be directly related to the size of the plumes. The comparison with observed splitting coefficients can then be done by computing the χ^2 misfit as:

$$\chi^2(x, y, z) = \sum(C_{obs} - C_{mantle}(x, y, z))^2/\sigma^2_{obs},$$

where C_{obs} and σ^2_{obs} are the observed degree two interaction coefficients and their associated errors for 28 mantle modes given by *Ritzwoller et al* [1988]. The interaction coefficient C_{mantle} can be computed from the splitting matrix (4) [*Giardini et al*, 1988], under the assumption of isolated multiplets, which is sufficient for eigenfrequency observations. Figure (1) shows the results if the value of z is 1. We see that the introduction of plumes in the lower mantle allows us to reach more than 80% variance reduction with very few additional parameters, and with a lower mantle model obtained by a superposition of smooth lateral variations approximately two times bigger than those of L0256 and by a global distribution of plumes with a temperature contrast of 340K. If the 7 fundamental modes are not taken into account, the variance reduction reaches 75%, for an amplification of L0256 of 1.8 and for plumes of 650K, and if only the higher 14 harmonics are taken, L0256 is less amplified (1.5), and the temperature contrast is 550K. As

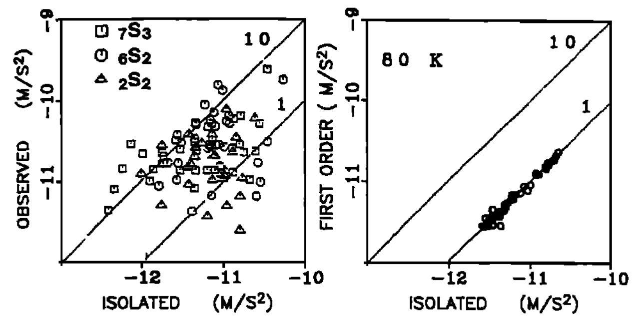

Fig. 3a-b. [a] Observed initial amplitudes of the core mode $_7S_3$ obtained by *Fukao & Suda* [1989]. The amplitudes are scattered compared to those computed for a spherical model. [b] First order computed initial amplitudes of the core mode $_7S_3$ for a plume model versus those computed in the isolated multiplet case for a temperature contrast of 80K.

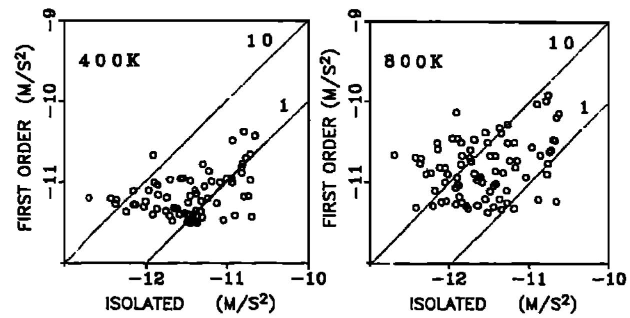

Fig. 3c-d. Same synthetic amplitudes as on figure 3b, but with a temperature contrasts of the plumes of 400K and 800K. The amplitude scattering is comparable to the one of *Fukao & Suda* for these temperature contrast and might be too large for 800K.

noted by *Ritzwoller et al* [1988], the apparently large amplification of L0256 can be reduced if one allows smooth aspherical boundary structure on the CMB and 670 km discontinuity. In figure 2, we now take exactly the smooth Vp structure of L0256, but allow the z parameter to vary in the lower mantle. For the 28 mantle mode coefficients, the best variance reduction is now obtained for plumes with a temperature contrast of 200K and for a scaling relation between V_p and V_s, ρ, 3 times larger than usual. Note that such values were obtained by *Giardini et al* [1988] and *Ritzwoller et al* [1988]. In all cases, the variance reduction is 10 to 15% better by adding plumes than without them. The obtained temperature contrast is close to the one obtained in convection simulations, and the mean ratio between the degree two patterns of L0256 and of plumes is 20%, which is of the same order of magnitude as the ratio expected from topographic swell observations [*Cazenave et al*, 1989]. But our results so far cannot resolve uniquely the size of small scale heterogeneities. We shall show in what follows, how amplitude data can be used for this purpose.

Constraining small scales heterogeneities

We have seen in the previous section that the splitting data are unable to separate smooth structure of the lower mantle, such as a large scale convective pattern, from the small part of a global distribution of short scale heterogeneities. Nevertheless, due to selection rules, only the structure with small scale heterogeneities, and not a smooth convective pattern, couples the fundamental and low angular order overtone modes with close eigenfrequencies and generally far apart in angular order. Recently, *Fukao & Suda* [1989] have observed the core modes $_2S_2$, $_6S_2$ and $_7S_3$, with associated eigenfrequencies of .95, 2.44 and 3.12 mHz, very close to some fundamental modes. They have noted that the initial amplitude of these modes shows significant scatter and cannot be explained by the theoretical amplitude obtained with a spherical model (figure 3a). Let us take the case of the most scattered core mode $_7S_3$ whose eigenfrequency differs from that of the fundamental $_0S_{22}$ by 7 μ Hz and assume that some heterogeneities can couple the two modes. First order perturbation theory can help us compute this coupling effect, which amounts to multiplying the spherical amplitudes A_m of each singlet of $_7S_3$ by:

$$\Big(1+\sum_{m'=-22}^{m'=22} \frac{R^{m'}_{{}_0S_{22}}}{R^{m}_{{}_7S_3}} \frac{H^{m'{}_0S_{22}}_{m{}_7S_3}}{\omega^2_{{}_7S_3}-\omega^2_{{}_0S_{22}}}\Big)\Big(1+\sum_{m'=-22}^{m'=22} \frac{S^{m'}_{{}_0S_{22}}}{S^{m}_{{}_7S_3}} \frac{H^{m'{}_0S_{22}}_{m{}_7S_3}}{\omega^2_{{}_7S_3}-\omega^2_{{}_0S_{22}}}\Big) \qquad (5)$$

where R and S are the isolated multiplet expressions of the receiver and source operators of the singlet m. With the help of relation (5), we see that: due to selection rules, the coupling is produced by lateral heterogeneities with degrees 19, 21, 23 and 25; such heterogeneities must be located in the mantle, where the amplitudes of the two modes are non zero; the coupling effect on the amplitude is amplified by the ratio of the spherical amplitude of $_0S_{22}$ over that of $_7S_3$, that is by an order of 10^2 or 10^3; such coupling effects are extremely sensitive to the frequency difference and can be computed only if perturbations are computed with respect to a spherical anelastic reference model where the mean (or observed) anelastic eigenfrequencies of both modes are used in (5) instead of the elastic, real ones. The two first conditions are verified by any global distribution of small scale heterogeneities in the mantle with a characteristic length smaller than a thousand kilometers. Assuming that the scatter in amplitudes is related to a plume distribution, let us now see if a realistic plume structure is strong enough to produce such scatter by computing the hybrid mode $_7S_3$ coupled with all neighboring modes $_2S_{15}$, $_0S_{22}$, $_3S_{10}$, $_2T_9$, $_6S_4$, $_1T_{13}$, $_0T_{23}$, $_1S_{15}$ and $_0S_{23}$. Figure (3b) shows, for IDA stations and for the same earthquakes as *Fukao & Suda* [1989], the synthetic initial amplitudes with plumes coupling versus those computed without coupling. Here the plumes have a temperature contrast of only 80K for the usual scaling relation between V_p, V_s and ρ heterogeneities and no amplitude scatter is modeled. With the increase of the temperature contrast, the amplitude scatter, as shown in figure (3c) and (3d) for contrasts of 400K and 800K respectively, appears to be comparable to the one observed by *Fukao & Suda* [1989]. Note that if the upper mantle plume structure is taken into account, the temperature must be increased by 1/3, as the contribution of the upper mantle plumes (with 50 km radius) is opposite in sign to that of the lower mantle.

Discussion

We have shown that small scale heterogeneities, unresolved by tomographic lower mantle models, may contribute to the splitting and amplitude scatter of the core and mantle modes of the Earth and produce deviation patterns of the frequencies similar to the observed ones. If splitting and amplitude data are used, it is then possible to separate the low pass filtered global distribution of small scale heterogeneities from that induced by the global convective pattern of the Earth. In this paper, using a very simple model of mantle plumes, we have thus obtained temperature contrasts of the same order of magnitude with the two kinds of data, comparable with those of numerical simulation of plumes. Assuming that plumes are the most representative small scale heterogeneities of the lower mantle, this would lead to the conclusion that around 20% of the lower mantle degree two can be associated with a global distribution of plumes and seems to be unresolved by the tomographic model L0256. However, many other small scale heterogeneities, for example subducted slabs possibly unresolved by the upper mantle models or other plume models, can affect in the same way both splitting and amplitudes of core and mantle modes. Also the results presented here are based on models and measurements available in the literature, which may need to be modified in the future. It remains that splitting and amplitude data must then be used simultaneously in order to obtain information about the small scale heterogeneities of the Earth's mantle and their trade-off with global convective patterns.

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P. Lognonné and B. Romanowicz, Institut de Physique du Globe, 4 Place Jussieu, Tour 14, 75252 Paris Cedex 05, France.

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